Improved Withdrawal Theories for Cylinders by a More Accurate Description of Curvatures of Static Menisci

JOHN A. TALLMADGE

Drexel Institute of Technology, Philadelphia, Pennsylvania

We are concerned here with the prediction of entrainment in withdrawal geometries (*6*). Solutions of this problem have involved film curvatures for two types of menisci, static and dynamic.

STATIC MENISCUS CURVATURE

The LaPlace equation for static menisci, $(\Delta P/\sigma) = (1/R_1) + (1/R_2)$, provides important boundary conditions in fluid mechanics. One such condition is the top curvature C_S of a static meniscus on the outside of vertical cylinder. Consider a cylinder of radius R and the nondimensional radius $G \equiv R(\rho g/2\sigma)^{1/2}$. The theoretical equation for the curvature at the top of static meniscus has been shown to be

$$C_S = B + \frac{1}{2G} \tag{1}$$

Here B is the normalized meniscus height ($B \equiv b/a$), where b and a are the dimensional meniscus heights on a cylinder and a flat surface, respectively. The value a is equal to $(2\sigma/\rho g)^{1/2}$ and is sometimes called the capillary height. The term C_S in Equation (1) was formed by dividing the cylinder curvature by the flat plate curvature; as reported earlier (*8*), the curvature values are $2B + (1/G)$ and 2 respectively. Thus C_S equals unity for a flat plate ($G \to \infty$ and $B \to 1$).

Equation (1) is the exact solution of the LaPlace equation for a zero contact-angle fluid on a vertical cylinder. For this contact angle, it has been shown theoretically (*8*) that height B is only a function of radius G. Therefore the top curvature C_S is only a function of radius G.

Analytical solution of the B function does not appear feasible, but numerical values at discrete values of G have been obtained. The shape of the tabular function has been represented approximately (*8*) by the empirical, analytical expression

$$\frac{B}{1-B} \equiv \phi \doteq \alpha G^{\beta} \tag{2}$$

Placing B from Equation (2) into Equation (1), one obtains

$$C_S \doteq \frac{\alpha G^{\beta}}{1 + \alpha G^{\beta}} + \frac{1}{2G} \tag{3}$$

The first evaluation of Equation (3) was based on numerical values of B reported for the G range of 0.03 to 3, for which α and β were found to be 2.4 and 0.85 (*8*). Thus

$$C_S^* \doteq \frac{2.4G^{0.85}}{1 + 2.4G^{0.85}} + \frac{1}{2G} \tag{4}$$

DYNAMIC MENISCUS CURVATURE (WITHDRAWAL)

Consider the vertical, steady state withdrawal (*6*) of a cylinder of radius R from a large bath of a wetting Newtonian liquid, where withdrawal occurs at a constant speed u. This is free or unobstructed coating. In the constant thickness region above the meniscus, the thickness of the entrained film will be h_0.

The top curvature in withdrawal (C_m) is obtained by transformation of the liquid-gas interfacial radius from the static value of R to the withdrawal value of $(R + h_0)$, as described previously (*9*). With the equivalent nondimensional transformation from G to GS, Equation (4) becomes

$$C_m^* \doteq \frac{2.4(SG)^{0.85}}{1 + 2.4(SG)^{0.85}} + \frac{1}{2(SG)} \tag{5}$$

where $S \equiv 1 + (h_0/R)$. The validity of the transformation from the static value to the withdrawal value is considered as verified by experimental data (*9, 10*).

The term C_m is the meniscus curvature for cylinder withdrawal. By definition, the C_m term reduces to unity at the large radii and large G characteristic of a flat plate. At the small radii and small G characteristic of very fine wires, C_m becomes very large.

WITHDRAWAL THEORY AND PURPOSE

The prediction of film thickness as a function of speed and radius (with the effects of fluid properties μ, ρ, σ and gravity g implied) is written in speed-explicit form as $u = u(h_0, R)$. Using nondimensional speed $N_{Ca} \equiv u(\mu/\sigma)$, the speed-explicit form has been written (*7*) as $N_{Ca} = \phi[D, C_m(S, G), Y(S), G]$. Here $D \equiv h_0(\rho g/\sigma)^{1/2}$ and $Y \equiv S^2 \ln S - 0.5(S^2 - 1)$.

The best theory to date for predicting film thickness in cylinder withdrawal is the following gravity-corrected theory; it has been verified, within experimental precision, for a wide range of speeds and fluids and for all radii tested (*10*).

$$N_{Ca} = 1.09\,[DC_m]^{3/2} + 2YG^2 \tag{6}$$

The general purpose of this note is to improve cylinder withdrawal theories by improving the function used to evaluate C_m in Equation (6). The specific purposes of this note are to present a more accurate expression for the C_m function of S and G, namely Equation (8), and to describe the development and properties of Equation (8).

A SECOND EXPRESSION FOR STATIC CURVATURE [EQUATION (7)]

The film thickness theory represented by use of Equation (5) in Equation (6) is sufficiently accurate to describe the entrained mass flowrate for at least a 10,000-fold range of N_{Ca}, within the experimental variation of 14% (*9, 10*). Recent studies (*3*) in the complete removal of shorts objects (*6*) indicate that withdrawal theory can be used in removal.

However, the removal studies (*3*) have shown that, because of sensitivity, a more precise form of withdrawal theory is needed for a reasonably accurate prediction of removal mass. Improved precision is also sought in order to provide better understanding of the deviations which occur in withdrawal at high speeds and high N_{Ca}.

These and other needs for improved precision led to reexamination of the approximation made in obtaining C_m^* Equation (5), namely description of B by the approximate ϕ function of Equation (2). A detailed study indicated that two small but noticeable inaccuracies were present in Equation (4). These errors were due to the presence of a numerical error in B values and use of an improper range of G.

The error in B values and corrected values are described in detail elsewhere (*1*). The error in B values, which was due to a typographical error in a numerical program used earlier (*8*), amounted to about 7% in the G region of interest in withdrawal. Comparison of the new B values with equivalent values obtained independently by another numerical technique (*2*) confirmed that the new values are accurate to at least four significant figures.

The error due to the previous use of the improper range of G has been discussed and minimized, as described elsewhere (*5*). The previous range used, namely G of 0.03 to 3 (*8*), was replaced by choosing the G range so as to minimize the error in curvature in Equation (3). Comparing the magnitude of each term in Equation (3), using $\alpha = 2.4$ and $\beta = 0.85$ as reasonable estimates, indicated that the region for which the ϕ correlation should be most precise occurs at G_0 of 1 to 30 and higher. A log-log plot of ϕ vs. G in this region indicated a β of unity and $\alpha = 3.36$. Thus the more accurate description of top curvature for a static menisci on a cylinder is (*5*)

$$C_S \doteq \frac{3.36G}{1 + 3.36G} + \frac{1}{2G} \tag{7}$$

Use of the new α and β values corrects the previous errors in Equation (4). As a check, C_S values obtained using empirical Equation (7) were compared with theoretical values of four to five place accuracy for a range of G (0.003 to 30). The new curvature values from Equation (7) were found to be accurate to within 0.5% for all G and, furthermore, to be accurate to within 0.1% for all $G < 0.02$ and all $G > 2$. The largest differences of 0.3 to 0.5% were noted at G from 0.07 to 0.7.

THE NEW DYNAMIC CURVATURE EXPRESSION [EQUATION (8)]

Using Equation (7) and the static to withdrawal transformation described above, the curvature for the top of a withdrawal meniscus is now given as

$$C_m \doteq \frac{3.36(SG)}{1 + 3.36(SG)} + \frac{1}{2(SG)} \tag{8}$$

Equation (8) is a new equation.

Comparison of C_m of Equation (8) with C_m^* of Equation (5) indicates that the old C_m^* value has errors of 2 to 6% in the GS range of 0.3 to 3 and errors of about 6% in the GS range of 3 to 30.

Use of the C_m of Equation (8) is recommended in all available continuous withdrawal theories for cylinders (*6*), including Equation (6) for Newtonian fluids (*7, 10*), the special-case Newtonian theories for low speeds (*9*), and the theory for non-Newtonian fluids (*4*). The effect of the improved precision should be most apparent at larger radii (larger G) and higher speeds (thicker films and larger S) or both (larger GS).

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Area-Free Mass Transfer Coefficients for Liquid Extraction in a Continuously Worked Mixer

R. B. KEEY and J. B. GLEN

University of Canterbury, Christchurch, New Zealand

Most reported data for continuous-phase mass transfer in agitated extraction vessels are expressed in terms of $K_c a$, the mass transfer coefficient per unit volume. Recently Schindler and Treybal (*17*) report some area-free coefficients for the continuous extraction of water-saturated ethyl acetate with water itself. The object of this paper is to describe other data for area-free coefficients that pertain to continuously worked extractors of similar design. The system, however, differs in that it is ternary: a solute is extracted from the dispersed phase into the continuous phase, both phases being presaturated with respect to the other nonsolute.

THEORY

In an earlier paper (*9*), we derived an expression for the continuous-phase mass-transfer coefficients by using Lin and coworkers' ideas (*12*) on turbulence damping. Near a mobile surface, where the energy loss through dissipation in the continuous-phase boundary layer may be less than near a rigid surface, Piterskikh's relationship may hold:

$$\nu_{\text{turb.}} = b\ \nu_c (y/\delta_0)^2 \tag{1}$$

where b is a coefficient of order 0.01 (*11*). A parallel argument leads to

$$K_c = \sqrt{C_{DF}}\ U\ N_{Sc}^{-1/2} \tag{2}$$

in which C_{DF} is the drag coefficient for the mobile drop and U is the relative velocity between the drop and the continuous phase. This slip velocity is widely fluctuating due to the turbulence created by the impeller. An expression for the maximum slip velocity has been derived by Levich (*11*) when the motion of the particle can be described in terms of self-preserving range of turbulence frequencies. Levich finds, when gravitational and supplemental reaction terms are neglected and entrainment is nearly complete, that

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J. B. Glen is with ICIANZ, Melbourne, Victoria, Australia.